GENERAL HULL FORM EQUATIONS

$$\text{Displacement to Length Ratio} = \frac{\text{Long Tons Displacement}}{(.01\ LWL)^3} = \frac{\Delta_{LT}}{(.01\ LWL)^3}$$

$$\text{Prismatic Coefficient} = \frac{\text{Cu. Ft. Displacement}}{\text{Midsection Area} \bullet LWL} = \frac{\nabla}{A_m \bullet LWL}$$

$$\text{Block Coefficient} = \frac{\text{Cu. Ft. Displacement}}{LWL \bullet WLB \bullet \text{Body Draft}} = \frac{\nabla}{L \bullet B \bullet D_B}$$

$$\text{Midsection Coefficient} = \frac{\text{Midsection Area}}{\text{WLBeam} \bullet \text{Body Draft}} = \frac{A_m}{WLB \bullet D_B}$$

$$\text{Prismatic Coefficient} = \frac{\text{Block Coefficient}}{\text{Midsection Coefficient}} = \frac{C_B}{C_M}$$

$$\text{Waterplane Coefficient} = \frac{\text{WP Area}}{LWL \bullet WLB}$$

These are all dimensionless coefficients of form, which therefore allow comparison with other boats, or even with models, independent of their differences in size.

COEFFICIENTS OF FORM

DISPLACEMENT TO LENGTH

Ranges for Δ/L (Displacement to Length) have been slowly re-defined over the years, tending toward lighter displacements within each range.

Sail

Light Multi - Hulls	40 - 50
Ultra - Light Racers	100 - 150
Light Racers	150 - 200
Light Cruisers & Offshore Racers	200 - 275
Medium Weight Cruisers	275 - 325
Heavy Cruisers	325 - 400

Power

Light Displacement	75 - 200
Medium Displacement	200 - 300
Heavy Displacement	300 - 400

Heavier displacement requires more power, more sail area, and is more comfortable.
Heavier displacement combined with smaller water plane area results in lower accelerations.

Lighter displacement requires less power, less sail area, and is less comfortable.
Lighter displacement combined with larger water plane area results in higher accelerations.

SPEED TO LENGTH RATIO

For a displacement boat, speed is a function of waterline length. The maximum theoretical speed is ordinarily assumed to be 1.34 times the square root of the LWL.

$$\text{Speed to Length Ratio} = \frac{\text{Velocity in Knots}}{\sqrt{\text{Waterline Length}}} = \frac{V}{\sqrt{LWL}}$$

The theoretical maximum SLR of 1.34 is the speed that a pair of wave crests can move through the water, which is assumed to apply to the bow and stern waves generated by the vessel's motion. Above this natural 'wave train' speed the boat is attempting to climb its own bow wave.

Boats having a low D/L can exceed this (due to lesser wave making). Boats with a high D/L may not achieve this theoretical maximum (greater wave making).

PRISMATIC COEFFICIENT

The prismatic coefficient is the ratio of actual underbody volume to the volume of a prism having a length equal to the DWL, and a section equal to the boat's maximum sectional area.

The prismatic coefficient provides an indication of the distribution of displacement. It is an indication of the fineness of the ends relative to the midsection of the hull. A low prismatic means there are fine ends and large mid-body. A high prismatic means there is more displacement distributed toward the ends.

Since the fullness or fineness of the ends has a large effect on wave making resistance, for any given speed to length ratio there is an ideal prismatic coefficient. As speed increases and the bow and stern waves rise, additional buoyancy in the ends becomes more favorable. Since racing yachts are more often driven at higher speeds, they benefit from a higher prismatic.

Optimum prismatic coefficient depends on the expected speed range.

- Up to SLR of 1 the least resistance is with a prismatic of .53.
- At SLR of 1.2 the ideal prismatic is .58.
- At SLR of 1.35 the ideal prismatic is .62.
- At SLR of 1.8 the ideal prismatic is .70.

Philips-Birt and others suggest that it is better to err on the high side in order to allow the least penalty at higher speed, since light wind sailing suffers less from a high prismatic than fast sailing suffers from a low prismatic.

Sail

Opinion varies regarding the range of ideal prismatic coefficients. Most sources suggest that for average conditions, the optimum prismatic is between .54 and .56.

- Light winds and calm water favor a lower range of about .53 to .55.
- Heavy winds and ocean sailing favor a higher range of about . 55 to .58.

A higher prismatic provides greater buoyancy in the ends and improves pitch dampening. An asymmetric water plane also improves pitch dampening, as does having some separation between the center of the water plane and the center of buoyancy.

Opinion also varies regarding whether the keel should be included when figuring the prismatic. It is generally accepted that if the keel contributes significantly to the displacement of the ends, it should be included. Conversely, the keel should be excluded when figuring the prismatic for a canoe body and fin keel, or where the keel is a distinct appendage.

With a full bodied sailboat, having an integral full keel that fairs gradually into the hull, the prismatic should include the keel.

With a well defined keel, whether full or not, if you figure the prismatic coefficient for the hull separately, you have a much better idea of the overall shape of the canoe body. In this case a value can be given for the prismatic both with and without the keel.

In either case, the keel *does* contribute to the dynamic displacement of the hull as it moves through the water.

Power

For **displacement** power boats the same optimum prismatic / speed ranges as for sailboats would apply except that it may be easier to predict the usual operating speed. For **planing** power boats, the optimum prismatic is higher, as follows:

- Typical planing power boats seem to favor a range of from .60 to .70.
- Semi-planing boats seem to favor a range of from .60 to .65.
- Full planing boats seem to favor a range of from .65 to .70.

Planing is more easily achieved with a higher prismatic (displacement distributed toward the ends). For planing boats many of the same considerations apply as for displacement hulls.

SIMILITUDE OF SCALE

Surface Area varies as the **Square** of the Dimensions. (l x w)
Volume (Displacement) varies as the **Cube** of the Dimensions. (l x w x h)
Stability varies as the **Fourth Power** of the Dimensions. (l x w x h x righting arm)

Wetted Surface varies as the **Square** of the Dimensions.
Resistance varies as the **Cube** of the Dimensions.
Speed varies as the **Square Root** of the Waterline Length.

Sail Area varies as the **Square** of the Dimensions. (l x w)
Wind Pressure varies as the **Square** of the Wind Speed.
Heeling Moment varies as the **Cube** of the Dimensions. (l x w x heeling arm)

GM varies as the **Cube** of changes in Beam Only.
GM varies **Inversely** with Changes in Displacement Only. (lowers B-M)

Stability varies as the **Fourth Power** of the Dimensions.
Stability varies as the **Square** of changes in Beam Only.
Stability varies **Directly** with changes in Length Only.
(if other dimensions remain same).